The effective neutrino charge radius

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Abstract. It is shown that at one-loop order a neutrino charge radius (NCR) may be defined, which is ultraviolet finite, does not depend on the gauge-fixing parameter, nor on properties of the target other than its electric charge. This is accomplished through the systematic decomposition of physical amplitudes into effective self-energies, vertices, and boxes, which separately respect electroweak gauge invariance. In this way the NCR stems solely from an effective proper photon-neutrino one-loop vertex, which satisfies a naive, QED-like Ward identity. The NCR so defined may be extracted from experiment, at least in principle, by expressing a set of experimental electron-neutrino cross-sections in terms of the finite NCR and two additional gauge- and renormalization-group-invariant quantities, corresponding to the electroweak effective charge and mixing angle.

PACS. 13.15.+g Neutrino interactions – 13.40.Gp Electromagnetic form factors

1 Introduction

It is a well-known fact that in non-Abelian gauge theories off-shell Green's functions depend explicitly on the gauge-fixing parameter. Therefore, the definition of quantities familiar from QED, such as effective charges and form-factors, is in general problematic. Such has been the case with the neutrino electromagnetic form-factor and the corresponding NCR. The calculational fact that, within the Standard Model, the (off-shell) one-loop $\gamma^*\nu\nu$ vertex (and the NCR obtained from it) is a gauge-dependent quantity has been established beyond any doubt in the seventies [1]. Based on this observation, it was concluded that "the NCR", which is the derivative at $q^2 = 0$ of the electromagnetic form-factor $F(q^2)$ extracted form this vertex, is not a physical quantity. Of course, if something is gauge-dependent it is not physical. But the fact that the off-shell vertex is gauge-dependent only means that it just does not serve as a reasonable definition of the NCR, it does not mean that an *effective* NCR cannot be encountered which satisfies *all* necessary physical properties, gauge-independence being one of them. Indeed, since then, several papers in the literature have attempted to find a *modified* vertex-like amplitude, leading to a consistent definition of the electromagnetic NCR (see [2] for an extended list of references). The common underlying idea in all such papers is to rearrange the Feynman graphs contributing to the scattering amplitude of neutrinos with charged particles, in an attempt to find a vertex-like combination that would satisfy all desirable properties. Of course, in doing so, a plethora of non-trivial physical constraints need to be satisfied. For example, one should not enforce gauge-independence at the expense of intro-

ducing target-dependence. Therefore, a definite guiding-principle is needed, allowing for the construction of physical sub-amplitudes with definite kinematic structure (i.e. self-ener- gies, vertices, boxes).

2 The pinch technique effective vertex

What has been accomplished recently in [3] (and some of the literature cited therein) is the proof that there exists a well-defined and finite effective three-point (vertex) Green's function, which has the following properties: (**i**) it is independent of the gauge-fixing parameter (ξ); (**ii**) it is ultra-violet finite; (**iii**) it satisfies a QED-like Ward-identity; (**iv**) it captures all that is coupled to a genuine ($1/q^2$) photon propagator; (**v**) it couples electromagnetically to the target; (**vi**) it does not depend on the $SU(2) \times U(1)$ quantum numbers of the target-particles used; (**vii**) it has a non-trivial dependence on the mass m_i of the charged isospin partner f_i of the neutrino in question; (**viii**) it contains only physical thresholds; (**ix**) it satisfies unitarity and analiticity; (**x**) it can be extracted from experiments.

The theoretical methodology allowing this physically meaningful definition is that of the pinch technique (PT) [4]. The PT is a diagrammatic method which exploits the underlying symmetries encoded in a *physical* amplitude such as an S-matrix element, in order to construct effective Green's functions with special properties. In the context of the NCR, the basic observation, already put forth in [2], is that the gauge-dependent parts of the conventional $\gamma^*(q)\nu\nu$, (to which the gauge-dependent NCR is associated) communicate and eventually cancel *algebraically* against analogous contributions concealed inside

the $Z^*(q)\nu\nu$ vertex, the self-energy graphs, and the box-diagrams (if there are boxes in the process), *before* any integration over the virtual momenta is carried out. For example, due to rearrangement produced by the systematic triggering of elementary Ward identities the gauge-dependent contributions coming from boxes are not box-like, but propagator or vertex-like. To understand how the topological modifications, which allow the communication between initially different graphs, come about, notice that, at one-loop level, *all* virtual longitudinal momenta (k) originating from tree-level gauge-boson propagators inside Feynman graphs trigger two elementary Ward identities, which furnish *inverse propagators*. The first reads

$$
\begin{aligned}
\not{k} P_L &= (\not{k}+ \not{p})P_L - P_R \not{p} \\
&= S_{f'}^{-1}(\not{k}+ \not{p})P_L - P_R S_f^{-1}(\not{p}) \\
&\quad + m_{f'} P_L - m_f P_R,
\end{aligned} \tag{1}
$$

where $P_{R(L)} = [1 + (-)\gamma_5]/2$ is the chirality projection operator and S_f is the tree-level propagator of the fermion f; f' is the isodoublet-partner of the external fermion f. The second relevant Ward identity reads

$$
(k+q)^\nu \Gamma_{\alpha\mu\nu}(q, k, -k-q) = t_{\alpha\mu}(q) - t_{\alpha\mu}(k), \tag{2}
$$

where $\Gamma_{\alpha\mu\nu}$ is the bare triple-gauge-boson vertex, and $t_{\mu\nu}(q) = q^2 g_{\mu\nu} - q_\mu q_\nu$. We emphasize that *all* gauge-dependent parts cancel exactly at the end of the pinching procedure, even in the presence of *non-vanishing fermion masses* m_f and $m_{f'}$, contrary to recent claims [5].

The new one-loop proper three-point function $\widehat{\Gamma}^\mu_{A\nu_i\bar{\nu}_i}$ satisfies the properties listed before. In particular, properties from (iv) to (vi) ensure that it is a photon vertex, uniquely defined in the sense that it is independent of using either weak isoscalar sources (coupled to the B-field) or weak isovector sources (coupled to W^0), or any charged combination. The NCR, to be denoted by $\langle r_{\nu_i}^2 \rangle$, is then defined as $\langle r_{\nu_i}^2 \rangle = 6(d\widehat{F}_{\nu_i}/dq^2)_{q^2=0}$; a straightforward calculation yields

$$
\langle r_{\nu_i}^2 \rangle = \frac{G_F}{4\sqrt{2}\,\pi^2}\left[3 - 2\log\left(\frac{m_i^2}{M_W^2}\right)\right], \tag{3}
$$

where $i = e, \mu, \tau$, the m_i denotes the mass of the charged iso-doublet partner of the neutrino under consideration, and G_F is the Fermi constant.

3 Measuring the effective NCR

After arriving at a physically meaningful definition for the NCR, the next crucial question is whether the NCR so defined constitutes a genuine physical observable. In the rest of this section we will briefly discuss the method proposed in [3] for the extraction of the NCR from experiment.

It is important to emphasize that measuring the entire process $f^\pm\nu \to f^\pm\nu$ does *not* constitute a measurement of the NCR, because by changing the target fermions $f^\pm$ one will generally change the answer, thus introducing a target-dependence into a quantity which (supposedly) constitutes an intrinsic property of the neutrino. Instead, what we want to measure is the target-independent

Standard Model NCR only, stripped of any contributions depending on the specific properties of the target (mass, spin, weak hypercharge), except its electric charge. Specifically, as mentioned above, the PT rearrangement of the S-matrix makes possible the definition of distinct, physically meaningful sub-amplitudes, one of which, $\widehat{\Gamma}^\mu_{A\nu_i\bar{\nu}_i}$, is finite and directly related to the NCR. However, the remaining sub-amplitudes, such as self-energy, vertex- and box-corrections, even though they do not enter into the definition of the NCR, still contribute numerically to the entire S-matrix; in fact, some of them combine to form additional physical observables of interest, most notably the effective (running) electroweak charge and mixing angle. Thus, in order to isolate the NCR, one must conceive of a combination of experiments and kinematical conditions, such that all contributions not related to the NCR will be eliminated.

Consider the elastic processes $f(k_1)\nu(p_1) \to f(k_2)\nu(p_2)$ and $f(k_1)\bar{\nu}(p_1) \to f(k_2)\bar{\nu}(p_2)$, where f denotes an electrically charged fermion belonging to a different isodoublet than the neutrino ν, in order to eliminate the diagrams mediated by a charged W-boson. The Mandelstam variables are defined as $s = (k_1 + p_1)^2 = (k_2 + p_2)^2$, $t = q^2 = (p_1-p_2)^2 = (k_1-k_2)^2$, $u = (k_1-p_2)^2 = (k_2-p_1)^2$, and $s+t+u = 0$. In what follows we will restrict ourselves to the limit $t = q^2 \to 0$ of the above amplitudes, assuming that all external (on-shell) fermions are massless. As a result of this special kinematic situation we have the following relations: $p_1^2 = p_2^2 = k_1^2 = k_2^2 = p_1 \cdot p_2 = k_1 \cdot k_2 = 0$ and $p_1 \cdot k_1 = p_1 \cdot k_2 = p_2 \cdot k_1 = p_2 \cdot k_2 = s/2$. In the center-of-mass system we have that $t = -2E_\nu E_\nu'(1 - x) \leq 0$, where E_ν and E_ν' are the energies of the neutrino before and after the scattering, respectively, and $x \equiv \cos\theta_{cm}$, where θ_{cm} is the scattering angle. Clearly, the condition $t = 0$ corresponds to the exactly forward amplitude, with $\theta_{cm} = 0, \quad x = 1$.

At tree-level the amplitude $f\nu \to f\nu$ is mediated by an off-shell Z-boson, coupled to the fermions by means of the bare vertex $\Gamma^\mu_{Zf\bar{f}} = -i(g_w/c_w)\gamma^\mu[v_f + a_f\gamma_5]$ with $v_f = s_w^2 Q_f - \frac{1}{2}T_z^f$ and $a_f = \frac{1}{2}T_z^f$.

At one-loop, the relevant contributions are determined through the PT rearrangement of the amplitude, giving rise to gauge-independent sub-amplitudes. In particular, the one-loop AZ self-energy $\widehat{\Sigma}^{\mu\nu}_{AZ}(q^2)$ obtained is transverse, for *both* the fermionic and the bosonic contributions, i.e. $\widehat{\Sigma}^{\mu\nu}_{AZ}(q^2) = (q^2 g^{\mu\nu} - q^\mu q^\nu)\widehat{\Pi}_{AZ}(q^2)$. Since the external currents are conserved, from the ZZ self-energy $\widehat{\Sigma}^{\mu\nu}_{ZZ}(q^2)$ we keep only the part proportional to $g^{\mu\nu}$, whose dimension-full cofactor will be denoted by $\widehat{\Sigma}_{ZZ}(q^2)$. Furthermore, the one-loop vertex $\widehat{\Gamma}^\mu_{ZF\bar{F}}(q, p_1, p_2)$, with $F = f$ or $F = \nu$, satisfies a QED-like Ward identity, relating it to the one-loop inverse fermion propagators $\widehat{\Sigma}_F$, i.e. $q_\mu \widehat{\Gamma}^\mu_{ZF\bar{F}}(q, p_1, p_2) = \widehat{\Sigma}_F(p_1) - \widehat{\Sigma}_F(p_2)$. It is then easy to show that, in the limit of $q^2 \to 0$, $\widehat{\Gamma}^\mu_{ZF\bar{F}} \sim q^2\gamma^\mu(c_1 + c_2\gamma_5)$; since it is multiplied by a massive Z boson propagator $(q^2 - M_Z)^{-1}$, its contribution to the amplitude vanishes when $q^2 \to 0$. This is to be contrasted with the $\widehat{\Gamma}^\mu_{A\nu_i\bar{\nu}_i}$,

which is accompanied by a $(1/q^2)$ photon-propagator, thus giving rise to a contact interaction between the target-fermion and the neutrino, described by the NCR.

We next eliminate the box-contributions, by means of the "neutrino–anti-neutrino" method. The basic observation is that the tree-level amplitudes $\mathcal{M}_{\nu f}^{(0)}$ as well as the part of the one-loop amplitude $\mathcal{M}_{\nu f}^{(B)}$ consisting of the propagator and vertex corrections (namely the "Born-improved" amplitude), are proportional to $[\bar{u}_f(k_2)\gamma_\mu(v_f + a_f\gamma_5)u_f(k_1)][\bar{v}(p_1)\gamma_\mu P_L v(p_2)]$, and therefore transform differently than the boxes under the replacement [6] $\nu \rightarrow \bar{\nu}$, since[1]

$$\bar{u}(p_2)\gamma_\mu P_L u(p_1) \rightarrow -\bar{v}(p_1)\gamma_\mu P_L v(p_2) = -\bar{u}(p_2)\gamma_\mu P_R u(p_1).$$
(4)

Thus, under the above transformation, $\mathcal{M}_{\nu f}^{(0)} + \mathcal{M}_{\nu f}^{(B)}$ reverse sign once, whereas the box contributions reverse sign twice. These distinct transformation properties allow for the isolation of the box contributions when the forward differential cross-sections $(d\sigma_{\nu f}/dx)_{x=1}$ and $(d\sigma_{\bar{\nu}f}/dx)_{x=1}$ are appropriately combined. In particular, the combination $\sigma_{\nu f}^{(+)} \equiv (d\sigma_{\nu f}/dx)_{x=1} + (d\sigma_{\bar{\nu}f}/dx)_{x=1}$ does not contain boxes, whereas the conjugate combination of cross-sections, $\sigma_{\nu f}^{(-)} \equiv (d\sigma_{\nu f}/dx)_{x=1} - (d\sigma_{\bar{\nu}f}/dx)_{x=1}$, isolates the contribution of the boxes.

Finally, a detailed analysis shows that in the kinematic limit we consider, the Bremsstrahlung contribution vanishes, due to a completely destructive interference between the two relevant diagrams corresponding to the processes $fA\nu(\bar{\nu}) \rightarrow f\nu(\bar{\nu})$ and $f\nu(\bar{\nu}) \rightarrow fA\nu(\bar{\nu})$. The absence of such corrections is consistent with the fact that there are no infrared divergent contributions from the (vanishing) vertex $\widehat{\Gamma}_{ZF\bar{F}}^\mu$, to be canceled against.

$\sigma_{\nu f}^{(+)}$ receives contributions from the tree-level exchange of a Z-boson, the one-loop contributions from the ultraviolet divergent quantities $\widehat{\Sigma}_{ZZ}(0)$ and $\widehat{\Pi}^{AZ}(0)$, and the (finite) NCR, coming from the proper vertex $\widehat{\Gamma}_{A\nu_i\bar{\nu}_i}^\mu$. The first three contributions are universal, i.e. common to all neutrino species, whereas that of the proper vertex $\widehat{\Gamma}_{A\nu_i\bar{\nu}_i}^\mu$ is flavor-dependent.

To proceed, the renormalization of $\widehat{\Sigma}_{ZZ}(0)$ and $\widehat{\Pi}_{AZ}(0)$ must be carried out. It turns out that, by virtue of the Abelian-like Ward-identities enforced after the pinch technique rearrangement [4], the resulting expressions combine in such a way as to form manifestly renormalization-group invariant combinations [7]. In particular, after carrying out the standard re-diagonalization, two such quantities may be constructed:

$$\bar{R}_Z(q^2) = \frac{\alpha_w}{c_w^2}\left[q^2 - M_Z^2 + \Re e\{\widehat{\Sigma}_{ZZ}(q^2)\}\right]^{-1}$$

$$\bar{s}_w^2(q^2) = s_w^2\left(1 - \frac{c_w}{s_w}\Re e\{\widehat{\Pi}_{AZ}(q^2)\}\right),$$
(5)

where $\alpha_w = g_w^2/4\pi$, and $\Re e\{...\}$ denotes the real part.

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[1] Equation (4) appears in [3] with an inconsequential sign error in the intermediate step.

In addition to being renormalization-group invariant, both quantities defined in (5) are process-independent; $\bar{R}_Z(q^2)$ corresponds to the Z-boson effective charge, while $\bar{s}_w^2(q^2)$ corresponds to an effective mixing angle. We emphasize that the renormalized $\widehat{\Pi}_{AZ}(0)$ cannot form part of the NCR, because it fails to form a renormalization-group invariant quantity on its own. Instead, $\widehat{\Pi}_{AZ}(0)$ must be combined with the appropriate tree-level contribution (which evidently does not enter into the definition of the NCR, since it is Z-mediated) in order to form the effective $\bar{s}_w^2(q^2)$ acting on the electron vertex, whereas the finite NCR will be determined from the proper neutrino vertex only.

After casting $\sigma_{\nu f}^{(+)}$ in terms of renormalization-group invariant blocks, one may fix $\nu = \nu_\mu$, and then consider three different choices for f: (i) right-handed electrons, e_R; (ii) left-handed electrons, e_L, and (iii) neutrinos, ν_i other than ν_μ, i.e. $i = e, \tau$. Thus, we arrive at the system

$$\sigma_{\nu_\mu\nu_i}^{(+)} = s\pi\bar{R}^2(0)$$

$$\sigma_{\nu_\mu e_R}^{(+)} = s\pi\bar{R}^2(0)\,\bar{s}_w^4(0) - 2\lambda s_w^2\left\langle r_{\nu_\mu}^2\right\rangle$$

$$\sigma_{\nu_\mu e_L}^{(+)} = s\pi\bar{R}^2(0)\left(\frac{1}{2} - \bar{s}_w^2(0)\right)^2$$
$$+\lambda(1 - 2s_w^2)\left\langle r_{\nu_\mu}^2\right\rangle$$
(6)

where $\lambda \equiv (2\sqrt{2}/3)s\alpha\,G_F$, $\alpha = e^2/4\pi$. $\bar{R}^2(0)$, $\bar{s}_w^2(0)$, and $\left\langle r_{\nu_\mu}^2\right\rangle$ are treated as three unknown quantities, to be determined from the above equations.

To extract the experimental values of the quantities $\bar{R}^2(0)$, $\bar{s}_w^2(0)$, and $\left\langle r_{\nu_\mu}^2\right\rangle$, one must substitute in the above equations the experimentally measured values for the differential cross-sections $\sigma_{\nu_\mu e_R}^{(+)}$, $\sigma_{\nu_\mu e_L}^{(+)}$, and $\sigma_{\nu_\mu\nu_i}^{(+)}$. Thus, one would have to carry out three different pairs of experiments.

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